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Numerical modeling of the plasma response to local cooling

Mikhail Z. Tokar and Mikhail Koltunov

Institute for Energy and Climate Research - Plasma Physics (IEK-4), Research Center Jülich,
52428, Jülich, Germany

E-mail: m.tokar@fz-juelich.de

Abstract. Thermal instabilities can lead to a sudden drop of local plasma temperature by orders of magnitude and numerical modeling of the global plasma response to such phenomena is performed. An approach to operate with the plasma heat conduction κ , being a non-linear function both of the plasma temperature and of its gradient, if the heat flux limit is taken into account, is developed. It is demonstrated that the deviation of κ from Spitzer-Härm approximation normally assumed is essential to explain experimental findings by massive gas injection experiments carried out to mitigate plasma disruptions in tokamaks.

1. Introduction

There are diverse causes for abrupt cooling of small localized regions in fusion and astrophysical plasmas. This can happen, e.g., by the penetration of impurity particles from intensive sources when electrons and ions in the background hydrogen plasma lose their thermal energy in elastic coulomb collisions and in inelastic excitation processes with impurities. Owing to thermal instabilities the local plasma temperature can drop by orders of magnitude during a very short time in a microsecond range [1]. Modeling of the plasma response to a localized cooling is essential for interpreting and predicting global consequences of such events. It is, however, a challenging numerical task due to some reasons. First, the problem is non-stationary and multi-dimensional. Second, the heat conduction κ is a strongly non-linear function of the plasma temperature T . In addition, because of the very sharp temperature gradient in the vicinity of the plasma area initially cooled down, the local Spitzer-Härm approximation for κ [2] fails and κ becomes depending on the temperature gradient due to heat flux limit phenomenon [3].

In this paper a numerical approach allowing coping with the problems outlined is presented. As the first step in the original heat transport equation we discretize the partial derivatives with respect to time and spatial variables except the one indicated as x henceforth. As a result one obtains coupled ordinary differential equations (ODE) of the second order for $T_j(x)$ with j counting the sets of discretized variables. These equations are solved in an iterative procedure by the method elaborated in Ref.[4] where in the vicinity of grid knots the ODE obtained are approximated by ODE with constant coefficients and exact analytical solutions of these equations are conjugated by requiring the continuity of the solution and its first x -derivative.

Computations are done for the conditions of massive gas injection into the tokamak JET performed to mitigate plasma current disruptions [5]. The radial transport across magnetic surfaces is approximated by a source term proportional to the deviation of the temperature

from that before the injection and non-stationary two-dimensional heat transfer on the surface is modeled. It is demonstrated that only if the deviation from the local Spitzer-Härm approximation to the heat conduction due to the heat flux limit is taken into account, calculations give a decay time of the temperature far from the injection position in an agreement with the experimentally measured level, in sub-millisecond range.

2. Basic equations and results of calculations

We proceed from a non-stationary heat balance equation in the coordinate system on the magnetic surface with the coordinates l and s aligned along and across the magnetic field, correspondingly:

$$3\partial_t(nT) + \partial_l q = \nu n(T_0 - T) \quad (1)$$

where n and T are the plasma density and temperature, respectively; $q = -\kappa_{||}\partial_l T$ is the parallel heat flux density with $\kappa_{||}$ being the plasma heat conduction along the magnetic field, and the source term in the right hand side characterizes the heat transport in the radial direction across the magnetic surfaces induced by the deviation from the temperature before the local cooling; the frequency ν is introduced in this study as a free parameter and the sensitivity of the results to it will be investigated. In spite of the fact that only one spatial coordinate is involved into equation (1), this advantage is fictitious since no boundary conditions can be posed to equation (1) in the toroidal geometry. Therefore one has to transfer to the toroidal and poloidal coordinates x and y with respect to which all physical quantities are periodic with periods $2L$ and 2Δ , respectively. The transformation between two coordinate system is straightforward, $l = x \cos \psi + y \sin \psi$, $s = -x \sin \psi + y \cos \psi$, where ψ is the pitch-angle of the magnetic field with respect to the toroidal direction. By using formulas for the differentiation of functions of many variables one gets instead of Eq.(1) the following one:

$$\begin{aligned} & 3\partial_t(nT) - \cos^2 \psi \partial_x (\kappa_{||} \partial_x T) - \sin \psi \cos \psi \partial_y (\kappa_{||} \partial_x T) - \\ & - \sin \psi \cos \psi \partial_x (\kappa_{||} \partial_y T) - \sin^2 \psi \partial_y (\kappa_{||} \partial_y T) \\ & = \nu n(T_0 - T) \end{aligned} \quad (2)$$

The time and y derivatives are discretized with finite increments τ and h_y , respectively, by taking into account that h_y can vary with y_j counted with the subscript $j_{\min} \leq j \leq j_{\max}$. In this study we assume the density n as constant in space and time. As a result an ordinary differential equation of the second order is obtained which governs the x dependence of $T_j \equiv T(t, x, y_j)$:

$$d_x^2 T_j + a \cdot d_x T_j = b T_j - f \quad (3)$$

with

$$\begin{aligned} a &= d_x \ln \chi_{||,j}, \quad b = \frac{3/\tau + \nu}{\chi_{||,j} \cos^2 \psi} + \left(1 + \frac{\chi_{||,j+1}/\chi_{||,j}}{1 + h_{y,j}/h_{y,j-1}} + \frac{\chi_{||,j-1}/\chi_{||,j}}{1 + h_{y,j-1}/h_{y,j}} \right) \frac{\tan^2 \psi}{h_{y,j} h_{y,j-1}} \\ f &= \frac{3T_{-,j}/\tau + \nu T_0}{\chi_{||,j} \cos^2 \psi} + \frac{\tan \psi}{h_{y,j-1} + h_{y,j}} \left\{ \begin{aligned} & \left(1 + \frac{\chi_{||,j+1}}{\chi_{||,j}} \right) \left(\frac{\tan \psi}{h_{y,j}} T_{j+1} + d_x T_{j+1} \right) + \\ & \left(1 + \frac{\chi_{||,j-1}}{\chi_{||,j}} \right) \left(\frac{\tan \psi}{h_{y,j-1}} T_{j-1} - d_x T_{j-1} \right) + \\ & + a (T_{j+1} - T_{j-1}) \end{aligned} \right\} \end{aligned}$$

where $\chi_{||} = \kappa_{||}/n$ and $T_{-,j} = T(t - \tau, x, y_j)$.

In the region $|x - L| \leq \lambda$ and $|y - \Delta| \leq \delta$ the plasma temperature is maintained at a very low level $T_c \ll T_0$ for any time $t \geq 0$; out of this area $T = T_0$ at $t = 0$. Due to the symmetry of the problem the computational domain can be reduced to $0 \leq x \leq L$ and $0 \leq y \leq \Delta$ where the boundary conditions for equations (3) are $d_x T_j(x=0, L) = 0$ for $y_j < \Delta - \delta$, and $d_x T_j(x=0) = 0, T_j(x=L) = T_c$ for $\Delta - \delta \leq y_j \leq \Delta$; $T_{j+1}(x) = T_{j-1}(x)$ and $d_x T_{j+1}(x) = d_x T_{j-1}(x)$ for $j-1 = j_{\min}$ and $j+1 = j_{\max}$ corresponding to $y_{j_{\min}} = 0$ and $y_{j_{\max}} = \Delta$, respectively. As the first approximation to $T_j(x)$ we assume $T_{-,j}(x)$ for all $j_{\min} \leq j \leq j_{\max}$. With $T_{-,j}(x)$ used for $T_{j+1}(x)$, $T_{j-1}(x)$ and by calculating $\chi_{||}$, since this is a function of the temperature, equations (3) are solved numerically by the method outlined in Ref.[4]. The found solutions are mixed with the previous approximation to T to get the next one. This procedure continues till the error of calculations becomes much smaller than the iteration mixing coefficient.

Figure 1 demonstrates the results of numerical calculations with the Spitzer-Härm approximation to $\chi_{||}$ [2], $\chi_{||}^{SH} [m^{-1}s^{-1}] \approx 10^{18} T^{2.5} [eV] / n [m^{-3}]$, where the parameter units are in square brackets. Computations have been done for the conditions of the massive gas injection to mitigate plasma disruption in the tokamak JET [5] with $L \approx 9.42m$, $\Delta \approx 3.14m$, $\psi = 0.1$, $\lambda/L = \delta/\Delta = 0.1L$, the initial plasma temperature at the radial position in question $T_0 = 250eV$, $T_c = 1.5eV$ and $n = 5 \cdot 10^{19} m^{-3}$. Different curves in figure 1a, demonstrating the time evolution of the temperature at $x = y = 0$, T_s , correspond to different magnitudes of ν ; figure 1b shows the final stationary temperature profile for $\nu = 10^3 s^{-1}$. These calculations were performed with the time step $\tau = 1\mu s$, spatial grid increments decreasing in geometrical progression by approaching to the cooled plasma region, $|L - x| \leq \lambda$ and $|\Delta - y| \leq \delta$, the iteration mixing coefficient equal 1 and the accuracy 10^{-10} at each time step. The correctness of the solutions has been proven by comparing the results obtained with different τ . In figure 1 one can see that in the final stationary state the temperature far away from the cooled area depends on the magnitude of the radial transport characteristic ν , varying less than by factor 2 when ν changes by an order of magnitude. On the contrary, the characteristic rate of the temperature decay is practically independent of ν . However, the characteristic transient time of $100\mu s$ is much smaller than that of $0.5ms$ found in experiments.

A possible reason for discrepancy above, between the calculated and experimental decay rates of the temperature on a magnetic surface locally cooled down by impurity injection, may be a violation of the Spitzer-Härm approximation to $\kappa_{||}$ in the region of extremely sharp temperature gradient. If the characteristic scale for the temperature change along field lines, $L_T = 1/|\partial_l \ln T|$, becomes much less than the mean free path length of electrons between coulomb collisions, $\lambda_c [m] \approx 10^{16} T^2 [eV] / n [m^{-3}]$, a significantly reduced free-streaming heat flux, $q = \xi n T^{3/2} / \sqrt{m}$, may be expected. Interpretation of laser fusion experiments [3], experiments with heating of magnetic islands in tokamaks [6], Fokker-Planck simulations of laser produced plasmas [7], modeling of tokamak scrape-off layer with strong recycling [8], has provided $0.002 \leq \xi \leq 0.1$. The fact that $\xi \ll 1$ is explained by non-local effects in collisionless plasma, reducing the perturbation in the distribution function caused by the temperature gradient, and braking of light electrons with the ambipolar electric field. A smooth transition between collisional and collisionless limits can be described by the formula [3]:

$$\chi_{||} \approx \chi_{||}^{SH} / \left(1 + \frac{3\lambda_c}{\xi L_T} \right) \quad (4)$$

One can see that this formula results in an even stronger reduction of the heat conduction near the cooled region as one expects from the temperature dependence of $\chi_{||}^{SH}$. This should result in an increase of the decay time and bring it closer to the experimental observations. However, the dependence of $\chi_{||}$ on the temperature gradient, prescribed by equation (4), may be a source of numerical troubles and a significant reduction of the iteration mixing coefficient, down to

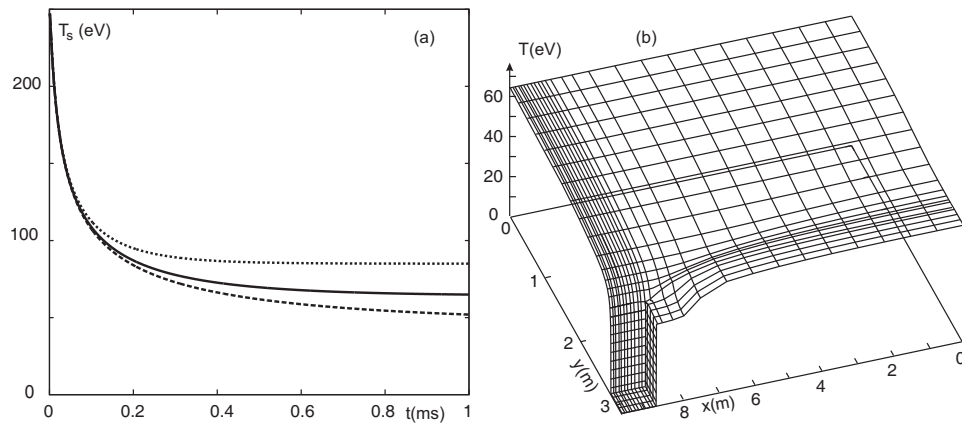


Figure 1. Time evolution of the plasma temperature far from the cooled area for $\nu = 1000$ (solid line), 300 (dashed line) and 3000 s^{-1} (dotted line) (a) and final stationary temperature profile (b) computed with Spitzer-Härm heat conduction.

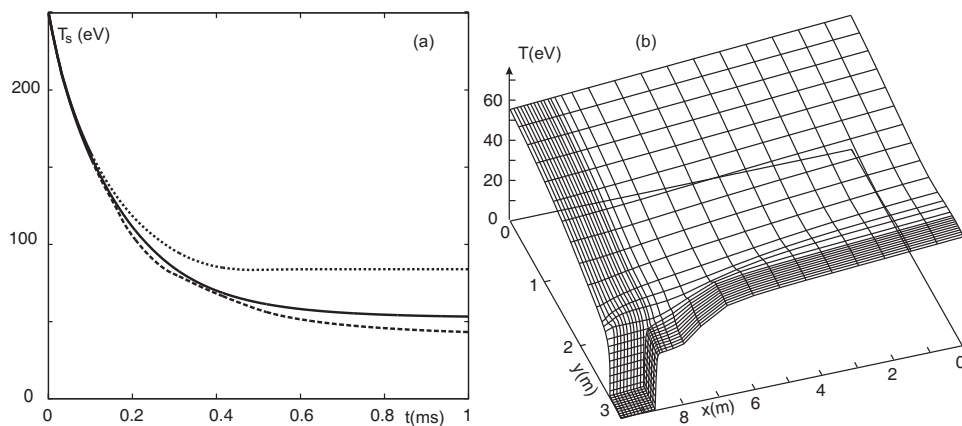


Figure 2. The same parameters as in figure 1 computed with the heat flux limit taken into account.

10^{-3} , is necessary to get converged solutions. This increases the computation time immensely. Figure 2 shows the time variation of T_s found with $\xi = 0.03$ and different magnitudes of ν and the temperature profile in the final stationary state for $\nu = 10^3 \text{ s}^{-1}$. One can see that with the deviations from the Spitzer-Härm approximation to the heat conduction, see equation (4), the decay time of the temperature far from the injection position agrees significantly better with the experimentally measured level [5] in the sub-millisecond range.

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